



## Modelling a Novel Multi-Objective Open-Shop Scheduling Problem and Solving by a Scatter Search Method

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### Keywords

Open shop scheduling problems,  
Tardiness and earliness time,  
Makespan, Setup cost, NSGA-II,  
Multi-objective scatter search

### ABSTRACT

*This paper proposes a novel, multi-objective integer programming model for an open-shop scheduling problem (OSSP). Three objectives are to minimize the makespan, total job tardiness and earliness, and total jobs setup cost. Due the complexity to solve such a hard problem, we develop a meta-heuristic algorithm based on multi-objective scatter search (MOSS), and a number of test problems are solved by this proposed algorithm. Finally, to prove its efficiency, the related results are compared with the results obtained by the well-known multi-objective evolutionary algorithm, called NSGA-II. The results confirm the efficiency and the effectiveness of our proposed MOSS to provide good solutions, especially for medium and large-sized problems.*

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NP-hard

NSGA-II

NSGA-II

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NSGA-II

NSGA-II

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$$[ \quad ] \quad .$$

PAES

SPEA

NSGA-II

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$$[ \quad ] .$$

NSGA-II

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Min $Z_1$	( )	•
Min $Z_2$	( )	
Min $Z_3$	( )	•
s. t.		•
$Z_1 \geq c(t, k)$	$\forall t, k$ ( )	•
$t(l, k) - t(l, k) + M(1 - a_{ilk})c(t, l) \geq$	( )	•
$c(j, k) - t(j, k) + M(1 - x_{ijk}) \geq c(t, k) \forall i, j, k$	( )	•
$a_{ilk} + a_{ikl} = 1$	$\forall i, k, l$ ( )	
$x_{ijk} + x_{jik} = 1$	$\forall i, j, k$ ( )	
$c(t, k) - t(t, k) \geq 0$	$\forall t, k$ ( )	$i=\{1, \dots, n\}$ : $j = i$
$mc(i) = \max\{c(t, k)\}$	$\forall t, k$ ( )	$m$ : $j=\{1, \dots, m\}$
$Z_2 = \sum_{i=1}^n \max\{mc(i) - d(i)\}$	( )	
$Z_3 = \sum_{k=1}^m \sum_{j=1}^n \sum_{i=1}^n s_i(j, k)x_{ijk}$	( )	
	( ) ( )	
	( )	
	Max	$k$ : $T_{ik}$
$Z_1$ $Z_1$		$i$ : $d_i$
		$k$ : $O_{ik}$
( ) ( )	Max	$j$ : $S_i(j, k)$
		$i$ : $k$
	( )	
		$k$ : $C_{ik}$
		$i$ : $mc_i$
( )		$\left. \begin{array}{l} 1 \text{ کار } i \text{ روی ماشین } k \text{ در صورتی که ماشین قبل } i \text{ باشد} \\ 0 \text{ در غیر این صورت} \end{array} \right\} a_{ilk}$
( )		$\left. \begin{array}{l} 1 \text{ کار } j \text{ روی ماشین } k \text{ در صورتی که کار قبل } j \text{ روی ماشین } k \text{ باشد} \\ 0 \text{ در غیر این صورت} \end{array} \right\} x_{ijk}$
	( )	
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*p*-medium

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b  
(3b-7)/2

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( )

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$n$

$m$

$m \times n$

( )

$k$

$i$

<sup>1</sup> $O_{12}$	<sup>2</sup> $O_{24}$		.....		.....	<sup><math>n \times m</math></sup> $O_{ik}$
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N

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$b_1$ )

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$b_2$

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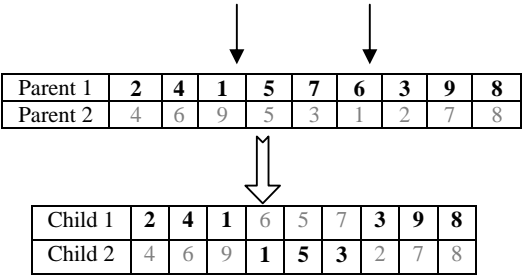
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N

i

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N :

2	5	3	6	4	1
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i

4	2	6	3	1	5
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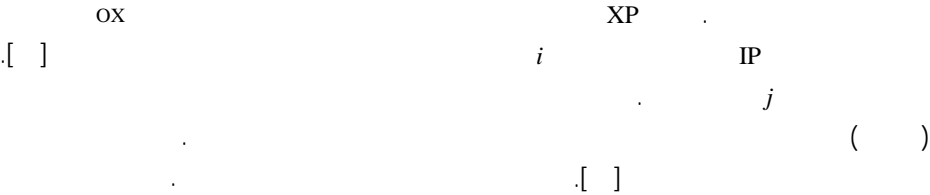
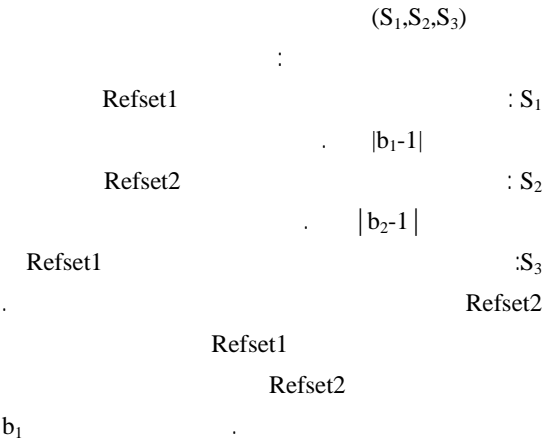
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$n \times m$

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4	2	6	3	1	5
1	2	6	3	4	5
5	2	6	3	4	1
2	5	6	3	4	1
2	5	3	6	4	1



Original trial solution 1 :  $1 \ 4 \ 7 \mid 2 \ 3 \ 5 \mid 9 \ 8 \ 6$

Original trial solution 2 :  $3 \ 9 \ 5 \mid 2 \ 4 \ 6 \mid 1 \ 7 \ 8$

( ) Refset2 ( ) Refset1

$|Refset|=b \leq b_1+b_2$

Refset1

$b_1$

$b_2 \quad b_1$

New trial solution 1 :  $9 \ 8 \ 6 \mid 1 \ 4 \ 7 \mid 2 \ 3 \ 5$

New trial solution 2 :  $1 \ 7 \ 8 \mid 3 \ 9 \ 5 \mid 2 \ 4 \ 6$

Refset1

$b_1$

New trial solution 1 :  $9 \ 8 \ 1 \ 7 \ 3 \ 5$

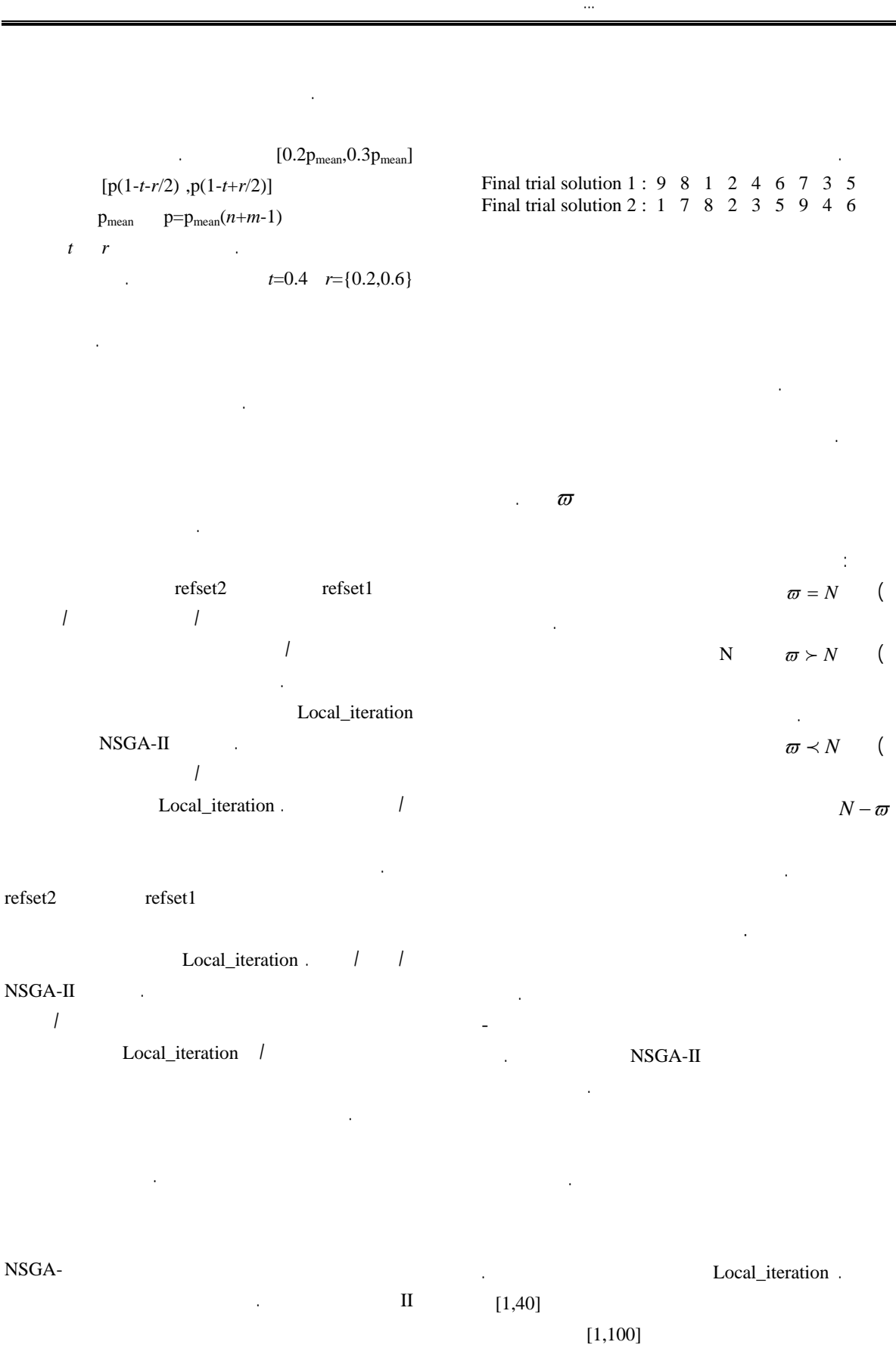
New trial solution 2 :  $1 \ 7 \ 8 \ 9 \ 4 \ 6$

Refset2

$b_2$  Refset2

Refset1

Refset1





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